

1 Rekurenté vzťahy

R1

$$a_{n+2} = 5a_{n+1} - 4a_n + 3; \quad a_0 = 1, a_1 = 3$$

R2

$$a_{n+2} = 5a_{n+1} - 4a_n + 3 \cdot n; \quad a_0 = 1, a_1 = 2$$

R3

$$a_{n+2} = 5a_{n+1} - 4a_n + 3 \cdot n - 2; \quad a_0 = 1, a_1 = 2$$

R4

$$2a_{n+2} = -3a_{n+1} + 2a_n + 4; \quad a_0 = 1, a_1 = 2$$

R5

$$2a_{n+2} = -3a_{n+1} + 2a_n - n; \quad a_0 = 1, a_1 = 2$$

R6

$$2a_{n+2} = -3a_{n+1} + 2a_n - n + 3; \quad a_0 = 1, a_1 = 2$$

R7

$$a_{n+2} = 3a_{n+1} - 2a_n - 3; \quad a_0 = 1, a_1 = 2$$

R8

$$a_{n+2} = 3a_{n+1} - 2a_n + 3; \quad a_0 = 1, a_1 = 2$$

R9

$$a_{n+2} = 3a_{n+1} - 2a_n + 3 + n; \quad a_0 = 1, a_1 = 2$$

R10

$$a_{n+2} = 3a_{n+1} - 2a_n + 3 - n; \quad a_0 = 1, a_1 = 2$$

2 Generujúce funkcie

GF1

$$a_{n+1} = a_n + \sum_{0 \leq k \leq n} a_k \quad a_0 = 1$$

GF2

$$a_{n+1} = F_n + \sum_{0 \leq k \leq n} a_k \quad a_0 = 1$$

GF3

$$a_{n+1} = F_n + \sum_{0 \leq k \leq n} a_k + n \quad a_0 = 1$$

GF4

$$a_{n+1} = \sum_{0 \leq k \leq n} a_k \cdot (n - k) \quad a_0 = 1$$

GF5

$$a_{n+1} = \sum_{0 \leq k \leq n} a_k \cdot F_{n-k} \quad a_0 = 1$$

GF6

$$a_{n+1} = \sum_{0 \leq k \leq n} a_k \cdot F_{n-k-1} \quad a_0 = 1$$

GF7 Vyjadrite v uzavretom tvare ogf pre

$$F_{3n+1}$$

GF8 Vyjadrite v uzavretom tvare ogf pre

$$F_{4n-3}$$

GF9 Vyjadrite v uzavretom tvare ogf pre

$$F_{6n+5}$$

GF10 Vyjadrite v uzavretom tvare ogf pre

$$F_{6n-1}$$

GF11 Vyjadrite v uzavretom tvare sumy

$$\sum_k \binom{n}{3k}$$

$$\sum_k \binom{n}{3k+1}$$

$$\sum_k \binom{n}{3k+2}$$

$$\sum_k \binom{n}{3k} (-1)^k$$

$$\sum_k \binom{n}{3k+1} (-1)^k$$

$$\sum_k \binom{n}{3k+2} (-1)^k$$

3 Asymptotické odhady

Odhadnite s požadovanou přesností:

A1

$$\binom{2n}{n}, \quad O(n^{-3})$$

A2

$$\binom{2n+1}{n-1}, \quad O(n^{-3})$$

A3

$$\frac{2n+1}{n-1}, \quad O(n^{-3})$$

A4

$$\left[\frac{2n+1}{n+1} \right]^{n-1}, \quad O(n^{-3})$$

A5

$$\sqrt{n^2-3} - \sqrt{n^2+5}$$

A6

$$\sum_{k=3}^{n-2} \binom{3n+1}{k-1}, \quad O(n^{-2})$$

A7

$$\sum_{k=0}^n \binom{5n}{2k}, \quad O(n^{-2})$$

A8

$$\sum_{0 \leq k \leq \lambda n} \binom{n}{k}, \quad \lambda < 1/2, \quad O(n^{-2})$$

A9

$$\sum_{0 \leq k \leq n} \frac{1}{n^2+k}, \quad O(n^{-7})$$

A10

$$\sum_{0 \leq k \leq n} \frac{1}{n^2+2k}, \quad O(n^{-3})$$