On $\mathbb{Z}$-flow-continuous maps between cubic graphs

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Abstract

Let M be an abelian group. An M-flow in an oriented graph $\vec{G}$ is an assignment $\phi\colon E(\vec{G}) \to M$ such that, at every vertex, the sum of all incoming flow values equals the sum of all outgoing flow values. A map $f\colon E(\vec{G}) \to E(\vec{H})$ between the edge sets of two oriented graphs is called M-flow-continuous if $\phi \circ f$ is an M-flow in $\vec{G}$ for every M-flow ϕ in $\vec{H}$. The Petersen Coloring Conjecture of Jaeger is equivalent to the statement that every bridgeless graph G admits a $\mathbb{Z}_2$-flow-continuous map to the Petersen graph.

The existence of M-flow-continuous maps naturally defines a quasi-order $\succ_M$ on the class of finite graphs. In 2017, Šamal proved that the quasi-order $\succ_{\mathbb{Z}_2}$ contains an infinite antichain of cubic graphs and asked whether an antichain can be found if we restrict to cyclically 4-edge-connected cubic graphs. It is known that $\mathbb{Z}$-flow-continuous maps are also $\mathbb{Z}_2$-flow-continuous. In this talk we present an explicit description of $\mathbb{Z}$-flow-continuous maps when restricted to cyclically 4-edge-connected cubic graphs and show that there is an infinite antichain of snarks in the quasi-order $\succ_{\mathbb{Z}}$.