

TOWARDS THE SMALLEST SIGNED PLANAR NON-4-COLOURABLE GRAPH

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In 2016, Máčajová, Raspaud and Škoviera conjectured that all signed planar graphs are signed 4-colourable. Three years later, Kardoš and Narboni found a counterexample, after having translated the problem of signed 4-colourability of a planar triangulation into an equivalent problem in the dual – the existence of a so-called consistent semi-2-factor (i.e., a collection of disjoint cycles covering all positive vertices, with every cycle containing an even number of them) in a vertex signed dual 3-connected cubic planar graph.

We study the properties of 3-edge-cuts and 4-edge-cuts with respect to consistent semi-2-factors in order to establish structural properties of a smallest counterexample. We classify 3-poles (graphs obtained by cutting along 3-edge-cuts) according to those properties into twelve classes, with the goal of searching exhaustively for a smallest representative of each class. We have developed a program that generates cubic planar graphs with specified structure (avoiding reducible configurations) and that verifies the existence of specific semi-2-factors. We have found the smallest representatives for six classes out of the twelve so far. The rest remains an open problem.