Some new types of graph regularities
and finite geometries

György Kiss
ELTE, Budapest (Hungary) & University of Primorska, Koper (Slovenia)

In this talk we consider some new types of graph regularities (edge-girth-regular, girth-regular, girth-biregular) and their connections to finite geometries.

Let Γ denote a simple, connected, finite k-regular graph. For an edge e of Γ let $n(e)$ denote the number of girth cycles containing e. For a vertex v of Γ let $\{e_1, e_2, \ldots, e_k\}$ be the set of edges incident to v ordered such that $n(e_1) \leq n(e_2) \leq \ldots \leq n(e_k)$. Then $(n(e_1), n(e_2), \ldots, n(e_k))$ is called the *signature* of v. The graph Γ is said to be *girth-(bi)regular* if (it is bipartite, and) all of its vertices (belonging to the same bipartition) have the same signature. Γ is called *edge-girth-regular*, if $n(e_1) = n(e_2) = \ldots = n(e_k)$ for all vertices.

We show that girth-(bi)regular graphs are related to (biregular) cages, finite projective and affine spaces and generalized polygons. We also present results in the spirit of stability theorems: we give upper bounds on $n(e_k) \leq M$ and show that if $n(e_k) = M - \epsilon$ for some non-negative integer ϵ, then $\epsilon = 0$.

Joint work with *Š. Miklavič* and *T. Szőnyi.*