Perfect versus imperfect matching covers of cubic graphs

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Given a bridgeless cubic graph G, the *four perfect matching defect* of G, denoted by $d_{PM}(G)$, is the minimum number of edges of G left uncovered over all sets of four perfect matchings. The *defect of a matching* is the number of vertices it leaves uncovered divided by two. The *four matching cover defect* of G, denoted by $d_M(G)$, is the minimal sum of defects of matchings over all matching covers of G containing four matchings.

We show that G always has a matching cover containing two perfect and two non-perfect matchings, and that G has a matching cover containing three perfect matchings and one non-perfect matching if and only if the Fan–Raspaud conjecture holds for G. We establish that $d_M(G) \leq d_{PM}(G)$. We show that for each integer k, there exists a nontrivial snark G with $d_M(G) = d_{PM}(G) = k$; that is, there are nontrivial snarks that are far from being coverable by four perfect matchings. Furthermore, we present another family of nontrivial snarks, where for each integer k there exists a nontrivial snark G with $2d_M(G) \leq d_{PM}(G) = 2k$. We conclude this talk with a discussion of open problems.

This is a joint work with Robert Lukoťka.