BRATISLAVA GRAPH THEORY SEMINAR

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Tweaking the Cartesian product of two cycles

Abstract: Let G be a graph admitting a perfect matching. A pairing of a graph G is a perfect matching of the complete graph on the same vertex set of G, and a graph is said to have the Pairing-Hamiltonian property if for every pairing of G there exists a perfect matching of G such that their union gives a Hamiltonian cycle. After having characterised which 3-regular graphs have the Pairing-Hamiltonian property, in [*Discrete Math. Theor. Comput. Sci.* **17(1)** (2015)], Alahmadi *et al.* suggest trying to tackle this characterisation problem for 4-regular graphs by looking at the Cartesian product of cycles $C_p \Box C_q$. We show that the only such graph admitting the Pairing-Hamiltonian property is $C_4 \Box C_4$, that is, the 4-dimensional hypercube. However, by slightly replacing some adjacencies in $C_p \Box C_q$ one can obtain what we define *accordion graphs*, which are 4-regular graphs on two parameters, n and k, denoted by $A[n, k]$. For every $n \geq 4$, $A[n, 2]$ has the Pairing-Hamiltonian property and empirical evidence suggests that, apart from $A[n, 2]$, there are (possibly an infinite number of) other accordion graphs having such a property or similar ones. Furthermore, once the accordion graph $A[n, 2]$ is drawn, one can immediately notice that it is a circulant graph. In [*Discrete Appl. Math.* **194** (2015)], Bogdanowicz gave a necessary and sufficient condition for a 4-regular circulant graph to be isomorphic to the Cartesian product of two cycles. In a similar way, we determine which accordion graphs are circulant, and also give all the possible values a 4-regular circulant graph can admit for it to be isomorphic to an accordion graph.

The above problems are joint work with John B. Gauci from L-Università ta' Malta.